

# Measuring Market Expectations in the Presence of Unobserved Fundamentals: A Predictive System for Exchange Rates

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## Abstract

In the presence of unobserved fundamentals, empirically modeling changes in the exchange rate as a linear function of observed fundamentals may result in spurious instances of the exchange rate disconnect puzzle. By treating the market expectation as a latent variable, we present a procedure for identifying and estimating the correlation between this latent variable and observed fundamentals, conditional on past information. A novel feature of the proposed model is the mechanism through which news on fundamentals leads to a revision of the market's expectations on future exchange rates. Empirical evidence suggests that the market expects a stronger home currency in response to news on higher relative inflation or output gap in the home country. The evidence is more compelling for the 1984Q1-2007Q2 period, during which the monetary policy is considered to be characterized by a Taylor rule, than for the sample that includes the post- Global Financial period. We also report a significant relationship between our measure of market expectations and survey forecasts of future exchange rate fluctuations.

**Key Words:** Predictive System, Exchange Rate Disconnect Puzzle, Latent Variable, Survey Forecasts, State-Space Model, News on Fundamentals.

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## 1. Introduction

As very well documented in Engel and West (2005) and Engel et al. (2007), one puzzle in international economics has been the divergence between theoretical models and empirical models of exchange rates, i.e., the exchange rate disconnect puzzle. Within the framework of a present value model, Engel and West (2005) show that the exchange rate follows a near random-walk process if fundamentals follow unit root processes and the factor for discounting future fundamentals is close to 1. They show that, in such cases, changes in the exchange rate may not be predictable by current fundamentals in the short run, even when the present value model is valid. This is because, in determining the exchange rate, current economic fundamentals have relatively little weight and much greater weight is put on expectations of future fundamentals when the discount factor is close to 1. Engel et al. (2007) thus argue that comparing the forecasting ability of a model relative to that of the random walk model may not be an appropriate way to evaluate an exchange rate model.

The challenge in evaluating economic models of the exchange rate is that not all the fundamentals are observable. Balke et al. (2013), for example, show that it is not possible to obtain sharp inference about the relative contribution of observed fundamentals in exchange rate fluctuations using only the data on observed fundamentals and exchange rates. Employing additional data on interest rate and price differentials, they find that unobserved fundamentals such as money demand shifts are an important contributor to exchange rate fluctuations. Evans and Lyons (2002) also show that failure of the exchange rate model results from omitting important unobserved fundamentals, by documenting that order flow, which conveys information not captured by observed macro variables, explains a substantial proportion of the fluctuation in the exchange rate. Bacchetta and van Wincoop (2004, 2013) theoretically show that the weakness of and instability in the relationship between exchange rates and observed fundamentals come from the sizable shocks to unobservable fundamentals that are ignored. Fratzscher et al. (2015) empirically confirm this.

In this paper, we present a framework for measuring the market expectation and for

uncovering the link between the exchange rate and observed fundamentals in the presence of unobserved fundamentals. When observed fundamentals are only a subset of exchange rate determination, the market expectation is no longer a linear combination of observed fundamentals. We follow Pastor and Stambuagh (2009) in treating the market expectation as a latent variable,<sup>2</sup> and present a procedure for estimating the correlation between this latent variable and observed fundamentals conditional on past information. A novel feature of this model is that it allows us to investigate a mechanism through which current news on observed fundamentals leads to a revision in the market participants' expectations on future exchange rate fluctuations.

Observed fundamentals we employ are the Taylor-rule fundamentals, i.e., the inflation and output gap differentials between two countries.<sup>3</sup> We use data on bilateral U.S. exchange rates versus those of Canada, Germany, Japan, and the United Kingdom over the 1984Q1 - 2007Q2 period, during which the monetary policy is considered to be characterized by the Taylor rule. Our empirical results suggest that there exists statistically significant nonlinear relationship between exchange rates and these fundamentals. In particular, we show that the market expects a stronger home currency in response to news on higher relative inflation or output gap in the home country. Furthermore, these news have persistent effects on the market expectation for most of the countries. When the sample is extended to include the post- Global Financial Crisis period, however, the evidence of the link between the market expectation and the Taylor-rule fundamentals is less compelling.

The outline of this paper is as follows. In Section 2, we first review the present value model of exchange rates by Engel and West (2005), who show that one source of the exchange rate disconnect puzzle is a factor for discounting future fundamentals being close to 1. We

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<sup>2</sup> Modeling expected returns as a latent variable is not new in the literature on the stock market. Refer to Brandt and Kang (2004) and Binsbergen and Koijen (2010).

<sup>3</sup> These variables have been the primary focus of recent literature in establishing the link between observed fundamentals and exchange rate movements. For example, Engel and West (2005) show that there exists Granger causality from exchange rates to these fundamentals. Based on high frequency data, Anderson et al. (2003), Faust et al. (2003, 2007), and Clarida and Waldman (2008) show that immediately after news indicating expansion in the U.S. economy or news about higher-than-expected inflation, an appreciation of the dollar follows. Engel and West (2006) and Mark (2009) also test the present value model of exchange rates by constructing model-based exchange rates based on VAR forecasts of the Taylor rule fundamentals and by comparing them with actual exchange rates.

then present an additional source of the exchange rate disconnect puzzle in the presence of unobservable fundamentals. In Section 3, we present a time series model of exchange rates in which the market expectation is treated as a latent variable. We then discuss a procedure for identification and estimation of the model. Section 4 provides empirical results, and Section 5 concludes the paper.

## 2. Potential Sources of the Exchange Rate Disconnect Puzzle

### 2.1. High Discount Factor: Engel and West (2005)

Engel and West (2005) provide a potential source of the exchange rate disconnect puzzle within the present value context. A version of the present value model considered by them is:

$$s_{t+1} = (1 - b) \sum_{j=0}^{\infty} b^j E_{t+1}(a' f_{t+j} + \tilde{a}' \tilde{f}_{t+j}), \quad 0 < b < 1, \quad (1)$$

where  $E_t(\cdot)$  refers to expectation conditional on information up to  $t$ ,  $b$  is a discount factor,  $f_t$  is a  $k \times 1$  vector of observable fundamentals, and  $\tilde{f}_t$  is a  $\tilde{k} \times 1$  vector of unobservable fundamentals. Assume, for simplicity, that  $\Delta f_t$  and  $\Delta \tilde{f}_t$  have the following Wold representations:

$$\Delta f_t = \theta_f(L) v_t, \quad (2)$$

$$\Delta \tilde{f}_t = \theta_{\tilde{f}}(L) \tilde{v}_t, \quad (3)$$

where  $\theta_f(L) = 1 + \theta_{f1}L + \theta_{f2}L^2 + \theta_{f3}L^3 + \dots$ , and  $\theta_{\tilde{f}}(L) = 1 + \theta_{\tilde{f}1}L + \theta_{\tilde{f}2}L^2 + \theta_{\tilde{f}3}L^3 + \dots$ . Then, following Engel and West (2005), we can derive a solution for  $\Delta s_{t+1}$ , given below: <sup>4</sup>

$$\Delta s_{t+1} = (1 - b) \sum_{j=0}^{\infty} b^j E_t(a' \Delta f_{t+j+1} + \tilde{a}' \Delta \tilde{f}_{t+j+1}) + a' \theta_f(b) v_{t+1} + \tilde{a}' \theta_{\tilde{f}}(b) \tilde{v}_{t+1}, \quad (4)$$

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<sup>4</sup> For derivation, refer to Appendix A.

where the first term on the right hand-side of equation (4) refers to the market expectation, as  $E_t(\Delta s_{t+1}) = (1 - b) \sum_{j=0}^{\infty} b^j E_t(a' \Delta f_{t+j+1} + \tilde{a}' \Delta \tilde{f}_{t+j+1})$ .

From equation (4), we have:

$$\lim_{b \rightarrow 1} \Delta s_{t+1} = a' \theta_f(1) v_{t+1} + \tilde{a}' \theta_{\tilde{f}}(1) \tilde{v}_{t+1}, \quad (5)$$

which suggests that, as the discount factor  $b$  approaches 1, the exchange rate approaches a random walk process. Intuitively, this is because current economic fundamentals have relatively little weight and much greater weight is put on the expectations of future fundamentals when the discount rate is close to 1. Thus, Engel and West (2005) suggest that a high discount factor is a potential source of the exchange rate disconnect puzzle. They test the validity of their model by showing that there exists a Granger causality from exchange rate to macro fundamentals.

## 2.2. Imposing a Linear Relationship Between Observed Fundamentals and the Market Expectation as an Additional Source

If we assume that  $\Delta f_t$  and  $\Delta \tilde{f}_t$  follow stationary VAR(1) processes, given by:

$$\Delta f_t = \Phi \Delta f_{t-1} + v_t, \quad \Delta \tilde{f}_t = \tilde{\Phi} \Delta \tilde{f}_{t-1} + \tilde{v}_t, \quad (6)$$

equation (4) can be rewritten as:

$$\Delta s_{t+1} = \beta_f \Delta f_t + \eta_{t+1}, \quad (7)$$

where  $\beta_f = (1 - b) a' (I_k - b\Phi)^{-1} \Phi$  and  $\eta_{t+1} = (1 - b) \tilde{a}' (I_{\tilde{k}} - b\tilde{\Phi})^{-1} \Delta \tilde{f}_t + a' (I_k - b\Phi)^{-1} v_{t+1} + b \tilde{a}' (I_{\tilde{k}} - b\tilde{\Phi})^{-1} \tilde{v}_{t+1}$ . When  $\Delta \tilde{f}_t$  (unobserved fundamentals) does not exist, we have  $\eta_{t+1} = a' (I_k - b\Phi)^{-1} v_{t+1}$  and equation (7) correctly specifies a linear relationship that exists between exchange rate and observed fundamentals. In this case,  $\eta_{t+1}$  is serially uncorrelated and not correlated with  $\Delta f_t$ . Thus, an OLS regression of equation (7) would allow us to efficiently estimate the market expectation.

When unobserved fundamentals play an important role in the exchange rate dynamics, however, OLS regression of equation (7) suffers from the omitted variables problem. If unobserved fundamentals are serially correlated and correlated with observed fundamentals, which may be usually the case,  $\Delta f_t$  may be correlated with  $\eta_{t+1}$  in equation (7). Thus, in the presence of unobserved fundamentals, estimating the market expectation by OLS regression of equation (7) may result in spurious instances of the exchange rate disconnect puzzle, as will be shown in Section 3.3.

### 3. Modeling the Market Expectation with Observed Fundamentals

#### 3.1. News on Observed Fundamentals and Revision in the Market Expectation: A Predictive System for Exchange Rates

In this section, instead of bypassing the existence of unobserved fundamentals  $\Delta \tilde{f}_t$  and estimating equation (7) by OLS, we treat the market expectation term in equation (4) as a latent variable and estimate it conditional on all available past information.<sup>5</sup> For this purpose, we rewrite equation (4) as:

$$\Delta s_{t+1} = \mu_t + u_{t+1}, \quad (8)$$

where  $\mu_t$  and  $u_{t+1}$  are the market expectation and the unexpected exchange rate, respectively, and these are given by:

$$\mu_t = E_t(\Delta s_{t+1}) = (1 - b) \sum_{j=0}^{\infty} b^j E_t(a' \Delta f_{t+j+1} + \tilde{a}' \Delta \tilde{f}_{t+j+1}) \quad (9)$$

and

$$u_{t+1} = a' \theta_f(b) v_{t+1} + \tilde{a}' \theta_{\tilde{f}}(b) \tilde{v}_{t+1}. \quad (10)$$

We assume that the latent variable  $\mu_t$  can be approximated by a stationary AR(p) process, given below:

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<sup>5</sup> When unobserved fundamentals  $\Delta \tilde{f}_t$  is serially uncorrelated and uncorrelated with observed fundamentals  $\Delta f_t$ , there would be no advantage in treating the market expectation as a latent variable. In this case, the market expectation estimated from the proposed model would be the same as that estimated from an OLS regression of equation (7).

$$\psi(L)\mu_t = \omega_t, \quad (11)$$

where  $\psi(L) = 1 - \psi_1 L - \psi_2 L^2 - \dots - \psi_p L^p$ . Note that the  $\omega_t$  term in equation (11) is the innovation to the market expectation or the conditional expectation of  $\Delta s_{t+1}$ . Assuming that both  $\Delta f_t$  and  $\Delta \tilde{f}_t$  follow VAR(1) processes as in equation (6), we can derive the following representation for the  $\omega_t$ :

$$\begin{aligned} \omega_t &= E_t(\mu_t) - E_{t-1}(\mu_t) \\ &= E_t(E_t(\Delta s_{t+1})) - E_{t-1}(E_t(\Delta s_{t+1})) \\ &= E_t(\Delta s_{t+1}) - E_{t-1}(\Delta s_{t+1}) \\ &= (1-b) \sum_{j=0}^{\infty} b^j (E_t - E_{t-1})(a' \Delta f_{t+j+1}) + (1-b) \sum_{j=0}^{\infty} b^j (E_t - E_{t-1})(\tilde{a}' \Delta \tilde{f}_{t+j+1}) \\ &= (1-b)a'(I_k - b\Phi)^{-1}v_t + (1-b)\tilde{a}'(I_{\tilde{k}} - b\tilde{\Phi})^{-1}\tilde{v}_t, \end{aligned} \quad (12)$$

where the two terms in the last line refer to news on observed fundamentals and news on unobserved fundamentals, respectively.

Equations (11) and (12) provide a mechanism through which news on fundamentals at time  $t$  leads to a revision of expectations in the current and subsequent periods. First, the  $\omega_t$  term in equation (12) shows how news on observed or unobserved fundamentals leads to a revision of the market expectation on  $\Delta s_{t+1}$ . Second, the autoregressive parameters in  $\psi(L)$  of equation (11) determine how the current news affects the market expectation on the subsequent exchange rates ( $\Delta s_{t+1+j}$ ,  $j = 1, 2, 3, \dots$ ). That is, current news on fundamentals can have persistent effects on the market expectation in subsequent periods.

In order to identify innovations to observed fundamentals, we assume the following stationary vector autoregressive process for observed fundamentals,

$$\Phi(L)(\Delta f_t - \alpha_f) = v_t, \quad v_t = A_v v_t^*, \quad v_t^* \sim i.i.d.N(0, I_k), \quad (13)$$

where  $\Delta f_t$  is a  $k \times 1$  vector and  $\Phi(L) = I_k - \Phi_1 L - \Phi_2 L^2 - \dots - \Phi_q L^q$ .

We can easily see from equation (12) that the link between the market expectation and observed fundamentals can be tested by the correlation between  $\omega_t$  in equation (11) and

$v_t$  in equation (13). Furthermore, there exists no a priori theory that restricts correlation between  $u_t$  and  $\omega_t$  or correlation between  $u_t$  and  $v_t$ . Thus, the distributional assumption for the innovations to the system (i.e., equations (8), (11), and (13)) is given by:

$$\begin{bmatrix} u_t \\ \omega_t \\ v_t \end{bmatrix} \sim i.i.d.N \left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{u\omega} & \Sigma_{uv} \\ \sigma_{\omega u} & \sigma_\omega^2 & \Sigma_{\omega v} \\ \Sigma_{vu} & \Sigma_{v\omega} & \Sigma_{vv} \end{bmatrix} \right), \quad (14)$$

where  $\Sigma_{vv} = A_v A_v'$ .

### 3.2. Measuring the Market Expectation: A Reduced-Form Model

Note that equations (8), (11), (13), and (14) form a version of Pastor and Stambaugh's (2009) predictive system, which they proposed to investigate the importance of a negative relationship between expected and unexpected stock returns on stock return predictability. As Pastor and Stambaugh (2009) mention, not all the correlation terms in equation (14) are identified.<sup>6</sup> However, we only need to identify the correlation between  $\omega_t$  and  $v_t$  in equations (11) and (13), in evaluating the link between the market expectation  $\mu_t$  and the observed fundamentals  $\Delta f_t$ . To achieve this goal, we consider the following three steps:

**Step 1:** We consider an orthogonal projection of  $\omega_t$  in equation (11) on  $v_t^*$  in equation (13):  $\omega_t = \gamma' v_t^* + \omega_t^*$ . This allows us to rewrite equation (11) as follows:

$$\psi(L)\mu_t = \gamma' v_t^* + \omega_t^*, \quad (15)$$

where  $\omega_t^*$  is not correlated with  $v_t^*$ .<sup>7</sup>

**Step 2:** We multiply both sides of equation (8) by  $\psi(L)$  to get:

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<sup>6</sup> Pastor and Stambaugh (2009) consider a predictive system for stock returns in which the fundamental is the price-dividend ratio. They solve the identification problem by employing a Bayesian approach. They find empirically that prior beliefs about the correlation between the innovation to expected returns and unexpected returns substantially affect estimates of expected returns as well as various inferences about predictability, including assessments of a predictor's usefulness.

<sup>7</sup> Here, the  $\omega_t^*$  term can be interpreted as the portion of the news on unobserved components that is orthogonal to the news on observed components.



$$\begin{aligned}\psi(L)\Delta s_{t+1} &= \psi(L)\mu_t + \psi(L)u_{t+1} \\ &= \gamma'v_t^* + \omega_t^* + \psi(L)u_{t+1}\end{aligned}\tag{16}$$

which can be rewritten as:

$$\psi(L)\Delta s_{t+1} = \gamma'v_t^* + \theta(L)e_{t+1}^*,\tag{17}$$

where  $\theta(L)e_{t+1}^* = w_t^* + \psi(L)u_{t+1}$ .

**Step 3:** We consider an orthogonal projection of  $e_{t+1}^*$  in equation (17) on  $v_{t+1}^*$  in equation (13):  $e_{t+1}^* = \delta'v_{t+1}^* + e_{t+1}$ . This allows us to rewrite equation (17) as follows:

$$\psi(L)\Delta s_{t+1} = \gamma'v_t^* + \theta(L)\delta'v_{t+1}^* + \theta(L)e_{t+1},\tag{18}$$

where  $e_{t+1}$  is not correlated with  $v_{t+j}^*$  for all  $j$ .

Then, equations (13) and (18) form our reduced-form model. The fact that  $e_{t+1}$  is uncorrelated with  $v_{t+j}$  for all  $j$  allows us to estimate equations (13) and (18) via a two-step procedure. That is, we can first estimate equation (13) by OLS and obtain  $\hat{v}_{t+1}^*$ . Then, in the second step, we can estimate equation (18) conditional on  $\hat{v}_{t+1}^*$ , using the maximum likelihood estimation method. In this two step procedure, however, we have to take care of the generated regressors problem when evaluating the standard errors of the parameter estimates in the second step regression. To avoid this problem, we can alternatively estimate equation (13) and (18) jointly. To jointly estimate these equations, we cast them into a state-space model and then apply the Kalman filter and the maximum likelihood estimation procedure to the state-space model.<sup>8</sup>

The link between the market expectation and observed fundamentals can then be tested by the statistical significance of the  $\gamma$  coefficients in equation (18), as  $\gamma$  is a linear function of the correlation between  $\omega_t$  and  $v_t^*$ . Note also that the Granger causality from observed fundamentals to exchange rates can be tested by the joint significance of the  $\gamma$  and  $\delta$  parameters. Once the parameters of the reduced-form model are estimated, the market expectation

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<sup>8</sup> For a state-space model for equations (13) and (18), readers are referred to Appendix B.

$\mu_t = E_t(\Delta s_{t+1})$ ,  $t = 0, 1, 2, \dots, T - 1$ , can be estimated by running the Kalman filter conditional on estimated parameters.

### 3.3. Performance of the Proposed Model in Comparison to OLS: Monte Carlo Experiment

In order to evaluate the performance of the reduced-form model presented in Section 3.2, we conduct Monte Carlo experiments in this section. We consider the following simplified data generating process, in which there exists nonlinear relationship between the market expectation and observed fundamentals:

#### Data Generating Process

$$\Delta s_{t+1} = \mu_t + u_{t+1}$$

$$\mu_{t+1} = \psi \mu_t + \omega_{t+1}$$

$$\Delta f_{t+1} = \phi \Delta f_t + \sigma_v v_{t+1}^*$$

$$\begin{bmatrix} u_{t+1} \\ \omega_{t+1} \\ v_{t+1}^* \end{bmatrix} \sim i.i.d.N \left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{u\omega} \sigma_u \sigma_\omega & \rho_{uv} \sigma_u \sigma_v \\ \rho_{u\omega} \sigma_u \sigma_\omega & \sigma_\omega^2 & \rho_{\omega v} \sigma_\omega \sigma_v \\ \rho_{uv} \sigma_u \sigma_v & \rho_{\omega v} \sigma_\omega \sigma_v & 1 \end{bmatrix} \right)$$

$$\psi = 0.95; \quad \sigma_v^2 = 0.1; \quad \sigma_\omega^2 = 0.048; \quad \sigma_u^2 = 1;$$

$$\rho_{uv} = 0.1; \quad \rho_{u\omega} = 0.3; \quad \rho_{\omega v} = 0.5.$$

We fix parameter values for  $\psi$ ,  $\sigma_u^2$ ,  $\sigma_\omega^2$ ,  $\sigma_v^2$ ,  $\rho_{uv}$ , and  $\rho_{u\omega}$  as given above. We then consider three alternative cases that differ in the values for the  $\phi$  parameter ( $\phi = 0.95$ ,  $0.5$ , or  $0.1$ ). We generate 5,000 sets of data for each of the three cases, and for each data set generated, we run the following regressions:

#### OLS with Observed Fundamentals

$$\Delta s_{t+1} = \beta \Delta f_t + \eta_{f,t+1}$$

Proposed Model (Reduced-Form Model)

$$\Delta s_{t+1} = \psi \Delta s_t + (\gamma - \theta \delta) v_t^* + \delta v_{t+1} + e_{t+1} - \theta e_t$$

$$\Delta f_{t+1} = \phi \Delta f_t + \sigma_v v_{t+1}^*,$$

where  $e_{t+1}$  is uncorrelated with  $v_{t+j}$  for all  $j$ .

Table 1 reports the mean and standard deviation of the  $R^2$  values, as well as those of the key parameter estimates from each regression. When we run an OLS regression using observed fundamentals, the results are discouraging. Even for the most favorable situation in which  $\phi = 0.95$ , the average  $R^2$  value is only 0.086. For the least favorable situation in which  $\phi = 0.1$ , the average  $R^2$  value decreases dramatically to 0.014. In this latter case, the mean for the estimates of  $\beta$  is not statistically different from zero. These results suggest that an OLS regression with observed fundamentals may result in spurious instances of the exchange rate disconnect puzzle in the presence of unobserved fundamentals.

In the last column of Table 1, we report results from the proposed reduced-form model. The parameters of the model are estimated with little bias. A very important finding is that the mean of the  $R^2$  values is around 0.204, regardless of the value of the  $\phi$  parameter. These simulation studies suggest that, unlike OLS regressions with observed fundamentals, our reduced-form model correctly identifies the nonlinear relationship that exists between exchange rate movements and observed fundamentals in the presence of unobserved fundamentals.

## 4. Uncovering the Relationship between Exchange Rates and the Taylor Rule Fundamentals: Empirical Results

### 4.1. Data Description

We study bilateral U.S. exchange rates versus those of the other four developed countries: Canada, Germany, Japan, and United Kingdom. Since Germany joined the European Monetary Union in 1999, implied rates for Germany mark is calculated from the official Euro conversion rate (1.95583 DEM/EUR). Observed fundamentals we employ are the Taylor-rule fundamentals, i.e., the inflation and output gap differentials between two countries. The International Financial Statistics (IFS) is the source for the end-of-quarter exchange rate  $s_t$ , seasonally-adjusted consumer price index  $p_t$ , and quarterly GDP  $y_t$ . To calculate the output gap,  $y_t^g$ , we consider percentage deviations of actual output from a quadratic time trend.<sup>9</sup> All data are converted by taking logs and multiplying by 100, so that their first differences are interpreted as percentage changes.

We employ a sample that covers the 1984Q1-2007Q2 period, during which the monetary policy is considered to be characterized by the Taylor rule. Note that the monetary policy might have changed since the 2007 Global Financial Crisis, as suggested by Molodtsova and Papell (2013) and Adrian, Etula, and Shin (2011). Thus, the link between the exchange rates and the Taylor rule fundamentals might have been weakened since 2007. To check this, we also employ an extended sample that ends in 2015Q4.

## 4.2. Estimation Results

We consider two alternative estimation strategies: i) OLS regressions of a linear model, in which  $\mu_t$ , the expected exchange rate change, is a linear function of the observed fundamentals, and ii) maximum likelihood estimation of a reduced-form model derived in Section 3.2. We assume that  $\mu_t$  in equation (11) follows an AR(2) process and that the vector of the Taylor rule fundamentals in equation (14) follow a VAR(2) process.<sup>10</sup> The linear model and the reduced-form model for our predictive system are described below:

### *Linear Model*

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<sup>9</sup> We also estimated the output gap by using the Hodrick and Prescott filter as an alternative, but the results were consistent.

<sup>10</sup> When there exists no Granger causality or feedback between the fundamentals, we employ a univariate AR(2) process for each fundamentals.

$$\Delta s_{t+1} = \beta_0 + \beta_p(\pi_t - \pi_t^*) + \beta_y(y_t^g - y_t^{*g}) + e_{t+1}^{ols}, \quad (19)$$

Proposed Model (Reduced-Form Model)

$$\begin{aligned} \Delta s_{t+1} = & \psi_1 \Delta s_t + \psi_2 \Delta s_{t-1} + \delta_p \theta_2 v_{p,t-1}^* + \delta_y \theta_2 v_{y,t-1}^* + (\gamma_p - \delta_p \theta_1) v_{pt}^* \\ & + (\gamma_y - \delta_y \theta_1) v_{yt}^* + \delta_p v_{p,t+1}^* + \delta_y v_{y,t+1}^* + e_{t+1} - \theta_1 e_t - \theta_2 e_{t-1}, \end{aligned} \quad (20)$$

$$(\Delta f_t - \alpha_f) = \Phi_1(\Delta f_{t-1} - \alpha_f) + \Phi_2(\Delta f_{t-2} - \alpha_f) + A_v v_t^*, \quad v_t^* \sim i.i.d.N(0, I_2), \quad (21)$$

where  $\Delta f_t = [(\pi_t - \pi_t^*) \quad (y_t^g - y_t^{*g})]'$  is a vector of observed fundamentals;  $v_t^* = [v_{y,t}^* \quad v_{p,t}^*]'$  is a vector of standardized residuals;  $\pi_t$  and  $y_t^g$  are inflation and the output gap for the home country, respectively (the foreign corresponding variables are denoted with an asterisk). Assuming symmetric Taylor rule coefficients in the home and foreign country, we use the inflation and the output gap differentials as observed fundamentals.

Table 2.1 reports results for the 1984Q1-2007Q2 sample. For the linear model, the null hypothesis that none of the Taylor rule variables have predictive power (i.e.,  $H_0 : \beta_p = \beta_y = 0$ ) is not rejected at the 5% significance level for any of the countries. For the proposed model, the estimates of the  $\gamma_p$  parameter are negative for all countries and those of the  $\gamma_y$  parameter are all negative except for U.K. A negative sign for  $\gamma_p$  or  $\gamma_y$  implies that the market expects the U.S. dollar (home currency) to appreciate in response to a news on relatively higher U.S. inflation or output gap. The null hypothesis that Taylor rule fundamentals have nothing to do with the market expectation (i.e.,  $H_0 : \gamma_p = \gamma_y = 0$ ) is rejected at the 2% significance level for all the countries under consideration. These results suggest that the market expects a stronger home currency in response to news on higher relative inflation or output gap in the home country. In Figure 1, we depict estimates of the market expectation obtained from both the linear model and our model against realized exchange rate fluctuations. We clearly see the differences in the dynamics of the expected exchange rate movements from the two alternative models. In particular, we can observe that the linear model cannot capture the dynamics of exchange rate fluctuations at a low frequency, while the proposed approach does.

Table 2.2 shows the results for an extended sample (1984Q1-2015Q4) that covers the post Global Financial Crisis period. Note that the relationship between the Taylor rule variables

and the market expectation from our model weakened considerably. For example, the null hypothesis that Taylor rule fundamentals have nothing to do with the market expectation (i.e.,  $H_0 : \gamma_p = \gamma_y = 0$ ) is not longer rejected at the 10% significance level for Canada or UK. Such results might be due to the monetary policy that changed with the onset of the Global Financial Crisis, as discussed in Adrian, Etula, and Shin (2011) and Molodtsova and Papell (2013).

### 4.3. Model-Based versus Survey-Based Measures of the Market Expectation

In this section, we examine whether the market expectation estimated from our model is related to the survey measure of the market expectation. The survey forecasts data is obtained by Economic Consensus Inc., and is sampled at a monthly frequency from October 1989 to December 2015 (from December 1994 to December 2015 for 24-month-ahead forecasts). We extract quarterly series from them. The survey is made by a large cross-section of professional market participants and reports the average forecasts of the spot exchange rate. To assess the association of the model-based and the survey-based measures of the Market expectation, we run the following regressions:

$$\text{Regression \#1 : } (s_{t+k}^F - s_t) = \beta_0 + \beta_1(E_t(s_{t+1}) - s_t) + e_t, \quad k = 1, 4, 8, \quad (22)$$

$$\text{Regression \#2 : } (s_{t+k}^F - s_t) = \beta_0 + \beta_1(E_t(s_{t+k}) - s_t) + e_t, \quad k = 4, 8, \quad (23)$$

where  $s_{t+k}^F$  is the k-quarter-ahead forecast from survey data and  $E_t(s_{t+k})$  is the k-quarter-ahead expected exchange rate estimated from our model. Note that  $E_t(s_{t+k}) - s_t$  is obtained by  $\sum_{j=1}^k E_t(\Delta s_{t+j})$  based on our model.

Table 3.1 and 3.2 show the results for the above two regressions. For short-horizon, we can see that the  $\beta_1$  coefficient is statistically significant at a 10% level for all countries except United Kingdom. In case of Canada and Japan, we can find the significant relation between survey data and our extracted expectation series even for the longer horizon. These results suggest that the market expectation estimated from our model with Taylor rule

fundamentals share common information with the actual market expectation obtained from the survey data.

## 5. Summary and Conclusions

As Engel and West (2005) show, a high discount factor in the presence of a unit root in fundamentals may serve as an important source of the exchange rate disconnect puzzle or the near random-walk behavior of the exchange rate reported in the literature. We first show that, in the presence of unobserved fundamentals, empirically modeling changes in the exchange rate as a linear function of observed fundamentals may result in spurious instances of the exchange rate disconnect puzzle. Then, by treating the market expectation as a latent variable as in Pastor and Stambaugh (2009), we present a procedure for identifying and estimating the correlations between this latent variable and the observed fundamentals, conditional on past information. A novel feature of the proposed model is that it provides a mechanism through which news on fundamentals at time  $t$  leads to a revision of expectations in the current and subsequent periods.

Our empirical results suggest that the market expects a stronger home currency in response to news on higher relative inflation or output gap in the home country. These results are obtained for the bilateral U.S. exchange rates versus those of Canada, Germany, Japan, and the United Kingdom over the 1984Q1 - 2007Q2 period, during which the monetary policy is considered to be characterized by a Taylor rule. However, the evidence of the link between the market expectation and these Taylor-rule fundamentals is less compelling for the sample that includes the period since the Global Financial Crisis. Besides, our empirical results suggest that there exists a significant relationship between our measure of market expectations and survey forecasts of future exchange rate fluctuations.

## Appendix A

Consider the following simplified version of the present value model investigated by Engel and West (2005). Here, for simplicity, we consider the case in which unobserved fundamentals do not exist. Then, the present value model of exchange rate is written as follow:

$$s_t = (1 - b) \sum_{j=0}^{\infty} b^j E_t[f_{t+j}], \quad 0 < b < 1, \quad (\text{A.1})$$

where  $f_t$  is a linear combination of observed fundamentals, and  $b$  is discount factor. Suppose further that there is a unit root in  $f_t$ , such that  $\Delta f_t$  has the following Wold representation:

$$\Delta f_t = \theta(L)\epsilon_t, \quad (\text{A.2})$$

where  $\epsilon_t$  is serially uncorrelated and  $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \theta_3 L^3 + \dots$ , with the roots of  $\theta(L) = 0$  lying outside the complex unit circle.

Then, we have the following expression for  $s_t$  and  $\Delta s_t$ :

$$\begin{aligned} s_t &= (1 - b) \sum_{j=0}^{\infty} b^j E_t[f_{t+j}] \\ &= \sum_{j=0}^{\infty} b^j E_t[f_{t+j}] - b \sum_{j=0}^{\infty} b^j E_t[f_{t+j}] \\ &= \sum_{j=0}^{\infty} b^j E_t[\Delta f_{t+j}] + \sum_{j=0}^{\infty} b^j E_t[f_{t+j-1}] - b \sum_{j=0}^{\infty} b^j E_t[f_{t+j}] \\ &= \sum_{j=0}^{\infty} b^j E_t[\Delta f_{t+j}] + f_{t-1} \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \Delta s_t &= \sum_{j=0}^{\infty} b^j E_t[\Delta f_{t+j}] - \sum_{j=0}^{\infty} b^j E_{t-1}[\Delta f_{t+j-1}] + \Delta f_{t-1} \\ &= \sum_{j=0}^{\infty} b^j E_t[\Delta f_{t+j}] - b \sum_{j=0}^{\infty} b^j E_{t-1}[\Delta f_{t+j}] \end{aligned} \quad (\text{A.4})$$

Due to law of iterative expectations, we have:

$$E_{t-1}[\Delta s_t] = (1 - b) \sum_{j=0}^{\infty} b^j E_{t-1}[\Delta f_{t+j}], \quad (\text{A.5})$$

and by combining equations (A.4) and (A.5), we have:



$$\begin{aligned}
\Delta s_t - E_{t-1}[\Delta s_t] &= \sum_{j=0}^{\infty} b^j E_t[\Delta f_{t+j}] - b \sum_{j=0}^{\infty} b^j E_{t-1}[\Delta f_{t+j}] - (1-b) \sum_{j=0}^{\infty} b^j E_{t-1}[\Delta f_{t+j}] \\
&= \sum_{j=0}^{\infty} b^j \{E_t[\Delta f_{t+j}] - E_{t-1}[\Delta f_{t+j}]\} \\
&= \sum_{j=0}^{\infty} b^j \theta_j \epsilon_t = \theta(b) \epsilon_t,
\end{aligned} \tag{A.6}$$

which allows us to rewrite  $\Delta s_t$  as:

$$\Delta s_t = (1-b) \sum_{j=0}^{\infty} b^j E_{t-1}[\Delta f_{t+j}] + \theta(b) \epsilon_t. \tag{A.7}$$

## Appendix B

In a simple case in which  $\mu_t$  follows an AR(1) process with  $\psi(L) = 1 - \psi L$  and  $\Delta f_{t+1}$  follows a VAR(1) process with  $\Phi(L) = I_k - \Phi L$ , the reduced-form model that consists of equations (11) and (16) can be cast into the following state-space model:

### Measurement Equation

$$\begin{bmatrix} \Delta f_{t+1} \\ \Delta s_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_f \\ 0 \end{bmatrix} + \begin{bmatrix} I_k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta f_{t+1}^* \\ \Delta s_{t+1}^* \\ e_{t+1} \\ v_{t+1}^* \end{bmatrix} \tag{B.1}$$

$$( Y_{t+1} = \tilde{\alpha} + H \xi_{t+1} )$$

### Transition Equation

$$\begin{bmatrix} \Delta f_{t+1}^* \\ \Delta s_{t+1}^* \\ e_{t+1} \\ v_{t+1}^* \end{bmatrix} = \begin{bmatrix} \Phi & 0 & 0 & 0 \\ 0 & \psi & -\theta & (\gamma' - \theta \delta') \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta f_t^* \\ \Delta s_t^* \\ e_t \\ v_t^* \end{bmatrix} + \begin{bmatrix} A_v & 0 \\ \delta' & 1 \\ 0 & 1 \\ I_k & 0 \end{bmatrix} \begin{bmatrix} v_{t+1}^* \\ e_{t+1} \end{bmatrix} \tag{B.2}$$

$$\left( \xi_{t+1} = F\xi_t + R\tilde{U}_{t+1}, \quad \tilde{U}_t \sim i.i.d.N(0, \Omega), \right)$$

where  $\Omega = \begin{bmatrix} I_k & 0 \\ 0 & \sigma_e^2 \end{bmatrix}$ .

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**TABLE 1. Performance of Alternative Models: Monte Carlo Experiments**

• **Data Generating Process:**

$$\begin{aligned} \Delta s_{t+1} &= \mu_t + u_{t+1} \\ \mu_{t+1} &= \psi \mu_t + \omega_{t+1} \\ \Delta f_{t+1} &= \phi \Delta f_t + \sigma_v v_{t+1}^* \\ \begin{bmatrix} u_{t+1} \\ \omega_{t+1} \\ v_{t+1}^* \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{u\omega} \sigma_u \sigma_\omega & \rho_{uv} \sigma_u \sigma_v \\ \rho_{u\omega} \sigma_u \sigma_\omega & \sigma_\omega^2 & \rho_{\omega v} \sigma_\omega \sigma_v \\ \rho_{uv} \sigma_u \sigma_v & \rho_{\omega v} \sigma_\omega \sigma_v & 1 \end{bmatrix} \\ & (\rho_{\omega v} = 0.5, \rho_{u\omega} = 0.3, \rho_{\omega v} = 0.5, \rho_{uv} = 0.1, \sigma_u^2 = 1, \sigma_\omega^2 = 0.048, \sigma_v^2 = 0.1) \end{aligned}$$

• **Estimation Methods:**

- Linear Model:  $\Delta s_{t+1} = \beta \Delta f_t + \eta_{t+1}^f$
- Proposed Model:

$$\begin{aligned} \Delta s_{t+1} &= \psi \Delta s_t + (\gamma - \delta \psi) v_t^* + \delta v_{t+1}^* + e_{t+1} - \theta e_t \\ \Delta f_{t+1} &= \phi \Delta f_t + \sigma_v v_{t+1}^* \end{aligned}$$

		True value	Linear Model	Proposed Model
$\phi = 0.95, \psi = 0.95$	$\beta$		0.326(0.185)	-
	$\gamma$	0.328	-	0.335(0.145)
	R <sup>2</sup>		0.086(0.075)	0.205(0.087)
$\phi = 0.50, \psi = 0.95$	$\beta$		0.459(0.265)	-
	$\gamma$	0.328	-	0.338(0.146)
	R <sup>2</sup>		0.027(0.025)	0.204(0.086)
$\phi = 0.10, \psi = 0.95$	$\beta$		0.358(0.248)	-
	$\gamma$	0.328	-	0.337(0.145)
	R <sup>2</sup>		0.014(0.014)	0.202(0.085)

Note: Sample size =250; Iterations =5,000. Standard deviations are reported in the parentheses.

**TABLE 2.1. Estimation of Models: Linear Model and Proposed Model (1984Q1-2007Q2)**

	Canada	Germany	Japan	United Kingdom
<b>Linear Model</b>				
$\Delta s_{t+1} = \beta_0 + \beta_p (\pi_t - \pi_t^*) + \beta_y (y_t^g - y_t^{g*}) + e_{t+1}^{ols}$				
$\beta_p$	-0.746 (0.721)	-0.026 (1.443)	-0.014 (1.523)	-2.406** (1.100)
$\beta_y$	-0.055 (0.059)	-0.168* (0.091)	-0.031 (0.061)	-0.013 (0.127)
R <sup>2</sup>	0.030	0.038	0.004	0.048
<i>Hypothesis Test</i> ( $H_0 : \beta_p = \beta_y = 0$ )				
p-value	0.272	0.174	0.867	0.083
<b>Proposed Model</b>				
$\Delta s_{t+1} = \psi_1 \Delta s_t + \psi_2 \Delta s_{t-1} + \theta_2 \delta_p v_{p,t-1}^* + \theta_2 \delta_y v_{y,t-1}^* + (\gamma_p - \theta_1 \delta_p) v_{pt}^* + (\gamma_y - \theta_1 \delta_y) v_{yt}^* + \delta_p v_{p,t+1}^* + \delta_y v_{y,t+1}^* + e_{t+1} - \theta_1 e_t - \theta_2 e_{t-1}$				
$(\Delta f_{t+1} - \alpha_\mu) = \Phi_1 (\Delta f_t - \alpha_\mu) + \Phi_2 (\Delta f_{t-1} - \alpha_\mu) + A_t v_{t+1}^*$ , where $\Delta f_{t+1} = [(\pi_{t+1} - \pi_{t+1}^*), (y_{t+1}^g - y_{t+1}^{g*})]'$ ,				
$v_{t+1}^* = [v_{y,t+1}^*, v_{p,t+1}^*]'$				
$\gamma_p$	-0.367* (0.209)	-0.177 (0.397)	-0.290*** (0.090)	-1.295** (0.516)
$\gamma_y$	-0.289* (0.172)	-0.953** (0.411)	-0.222*** (0.043)	0.118 (0.480)
$\delta_p$	-0.260 (0.326)	0.152 (0.669)	-1.869*** (0.658)	-0.692 (0.505)
$\delta_y$	0.172 (0.350)	0.122 (0.763)	-1.229** (0.597)	0.253 (0.501)
$\psi_1 + \psi_2$	0.903*** (0.078)	0.835*** (0.082)	0.983*** (0.009)	-0.056 (0.213)
$\theta_1 + \theta_2$	0.866*** (0.170)	0.815*** (0.087)	0.988*** (0.003)	0.045 (0.331)
R <sup>2</sup>	0.087	0.099	0.047	0.137
<i>Hypothesis Test</i> ( $H_0 : \gamma_p = \gamma_y = 0$ )				
p-value	0.007	0.004	0.001	0.016

Note: Standard errors are reported in the parentheses. Asterisks denote significance at levels 1%(\*\*\*) . 5% (\*\*). and 10% (\*), respectively.

**TABLE 2.2. Estimation of Models: Linear Model and Proposed Model (1984Q1-2015Q4)**

	Canada	Germany	Japan	United Kingdom
<b>Linear Model</b>				
$\Delta s_{t+1} = \beta_0 + \beta_p (\pi_t - \pi_t^*) + \beta_y (y_t^g - y_t^{g*}) + e_{t+1}^{ols}$				
$\beta_p$	0.161 (0.780)	-0.015 (1.225)	1.415 (1.096)	-0.672 (0.976)
$\beta_y$	-0.108* (0.064)	-0.132 (0.085)	-0.036 (0.057)	-0.050 (0.126)
R <sup>2</sup>	0.022	0.019	0.014	0.005
<i>Hypothesis Test</i> ( $H_0 : \beta_p = \beta_y = 0$ )				
p-value	0.237	0.300	0.418	0.675
<b>Proposed Model</b>				
$\Delta s_{t+1} = \psi_1 \Delta s_t + \psi_2 \Delta s_{t-1} + \theta_2 \delta_p v_{p,t-1}^* + \theta_2 \delta_y v_{y,t-1}^* + (\gamma_p - \theta_1 \delta_p) v_{pt}^* + (\gamma_y - \theta_1 \delta_y) v_{yt}^* + \delta_p v_{p,t+1}^* + \delta_y v_{y,t+1}^* + e_{t+1} - \theta_1 e_t - \theta_2 e_{t-1}$				
$(\Delta f_{t+1} - \alpha_\mu) = \Phi_1 (\Delta f_t - \alpha_\mu) + \Phi_2 (\Delta f_{t-1} - \alpha_\mu) + A_v v_{t+1}^*$ , where $\Delta f_{t+1} = [(\pi_{t+1} - \pi_{t+1}^*), (y_{t+1}^g - y_{t+1}^{g*})]'$ , $v_{t+1}^* = [v_{y,t+1}^*, v_{p,t+1}^*]'$				
$\gamma_p$	-0.181 (0.224)	-0.153 (0.302)	0.901* (0.481)	-0.003 (0.201)
$\gamma_y$	-0.036 (0.946)	-0.731** (0.330)	0.177 (0.406)	-0.016 (0.131)
$\delta_p$	0.201 (0.362)	0.511 (0.551)	-1.231** (0.553)	0.197 (0.436)
$\delta_y$	0.205 (0.437)	0.099 (0.530)	-0.475 (0.528)	-0.233 (0.436)
$\psi_1 + \psi_2$	0.950*** (0.106)	0.814*** (0.093)	0.442 (0.442)	0.791*** (0.076)
$\theta_1 + \theta_2$	0.999*** (0.055)	0.808*** (0.091)	0.275 (0.435)	0.999*** (0.065)
R <sup>2</sup>	0.060	0.061	0.084	0.112
<i>Hypothesis Test</i> ( $H_0 : \gamma_p = \gamma_y = 0$ )				
p-value	0.295	0.009	0.053	0.999

Note: Standard errors are reported in the parentheses. Asterisks denote significance at levels 1%(\*\*\*). 5% (\*\*), and 10% (\*), respectively



**TABLE 3.1. Relationship Between Survey-Based and Model-Based Measures of the Market Expectation (1989Q4-2007Q2)**

	Canada	Germany	Japan	United Kingdom
I. $s_{t+k}^F - s_t = \beta_0 + \beta_1(E_t(s_{t+1}) - s_t) + e_t$				
		<u>k=1</u>		
$\beta_1$	-0.378*** (0.141)	0.368** (0.145)	0.327* (0.176)	-0.029 (0.101)
R <sup>2</sup>	0.095	0.130	0.048	0.001
		<u>k=4</u>		
$\beta_1$	-0.735** (0.284)	0.197 (0.349)	0.582* (0.314)	0.160 (0.198)
R <sup>2</sup>	0.090	0.099	0.048	0.009
		<u>k=8</u>		
$\beta_1$	-0.760* (0.438)	0.390 (0.667)	0.830* (0.421)	-0.021 (0.376)
R <sup>2</sup>	0.059	0.068	0.075	0.000
II. $s_{t+k}^F - s_t = \beta_0 + \beta_1(E_t(s_{t+k}) - s_t) + e_t$				
		<u>k=4</u>		
$\beta_1$	-0.157* (0.081)	0.143 (0.105)	0.162** (0.080)	0.167 (0.209)
R <sup>2</sup>	0.052	0.107	0.057	0.009
		<u>k=8</u>		
$\beta_1$	-0.087 (0.068)	0.194 (0.133)	0.092* (0.054)	0.176 (0.367)
R <sup>2</sup>	0.033	0.089	0.057	0.005

Note: Standard errors are reported in the parentheses. Asterisks denote significance at levels 1%(\*\*\*), 5%(\*\*), and 10%(\*), respectively.  $s_{t+k}^F$  is the survey forecasts of k-quarter-ahead exchange rate, and  $E_t(s_{t+k})$  is the k-quarter-ahead market expectation of exchange rate fluctuations from our model. Note that

$E_t(s_{t+k}) - s_t = \sum_{j=1}^k E_t(\Delta s_{t+j})$ . The survey forecasts data are sampled at a monthly frequency from October

1989 to October 2014 (from December 1994 to October 2014 for 24-month-ahead forecasts), and we extract quarterly series from the monthly data. To match the date of the current spot exchange rate to that of the survey forecasts,  $s_t$  on the left hand-side of the equation is the spot rate which is collected at the date of survey forecasts. Because survey forecasts for the Germany mark changes to Euro from 1999Q1 and show large jump during this period, we drop the period 1998Q1-1999Q4 from the regressions for the Germany.

**TABLE 3.2. Relationship Between Survey-Based and Model-Based Measures of the Market Expectation (1989Q4-2015Q4)**

	Canada	Germany	Japan	United Kingdom
III. $s_{t+k}^F - s_t = \beta_0 + \beta_1(E_t(s_{t+1}) - s_t) + e_t$				
		<u>k=1</u>		
$\beta_1$	-0.315*	0.365**	-0.427**	0.182
	(0.162)	(0.165)	(0.131)	(0.277)
R <sup>2</sup>	0.36	0.096	0.094	0.004
		<u>k=4</u>		
$\beta_1$	0.066	0.220	-0.509*	0.288
	(0.323)	(0.354)	(0.263)	(0.498)
R <sup>2</sup>	0.000	0.113	0.035	0.003
		<u>k=8</u>		
$\beta_1$	0.787	-0.305	-0.524	2.148***
	(0.507)	(0.619)	(0.409)	(0.690)
R <sup>2</sup>	0.028	0.106	0.020	0.105
IV. $s_{t+k}^F - s_t = \beta_0 + \beta_1(E_t(s_{t+k}) - s_t) + e_t$				
		<u>k=4</u>		
$\beta_1$	0.303***	0.159	-0.174	0.036
	(0.106)	(0.107)	(0.168)	(0.161)
R <sup>2</sup>	0.073	0.116	0.010	0.001
		<u>k=8</u>		
$\beta_1$	0.908***	0.050	-0.228	0.361**
	(0.157)	(0.137)	(0.013)	(0.180)
R <sup>2</sup>	0.289	0.088	0.018	0.047

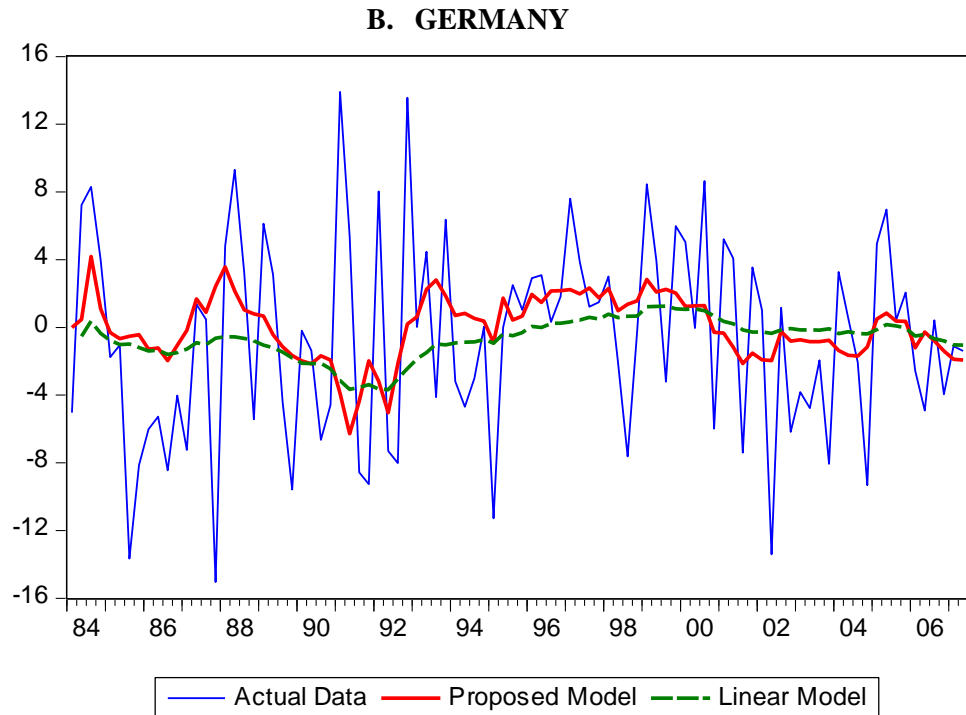
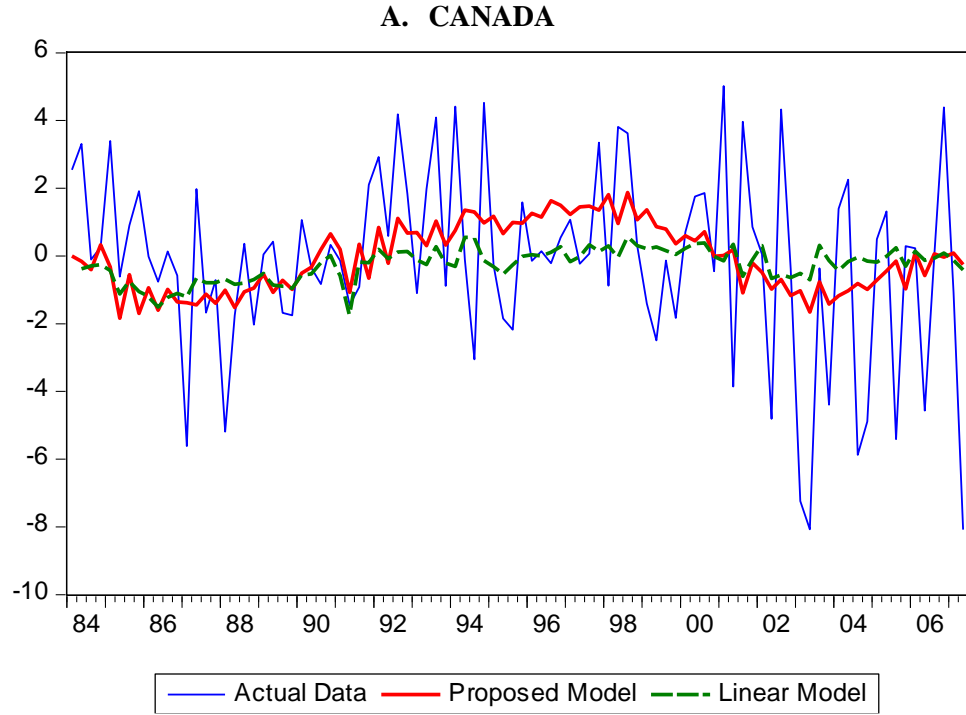
Note: Standard errors are reported in the parentheses. Asterisks denote significance at levels 1%(\*\*\*), 5%(\*\*), and 10%(\*), respectively.  $s_{t+k}^F$  is the survey forecasts of k-quarter-ahead exchange rate, and  $E_t(s_{t+k})$  is the k-quarter-ahead out-of-sample forecasts of exchange rate fluctuations from our model. Note that

$E_t(s_{t+k}) - s_t = \sum_{j=1}^k E_t(\Delta s_{t+j})$ . The survey forecasts data are sampled at a monthly frequency from October 1989 to December 2015 (from December 1994 to December 2015 for 24-month-ahead forecasts), and we extract quarterly series from the monthly data. To match the date of the current spot exchange rate to that of the survey forecasts, we use spot rate which is collected at the date of survey forecasts,  $s_t$ , on the left hand-side of the equation.

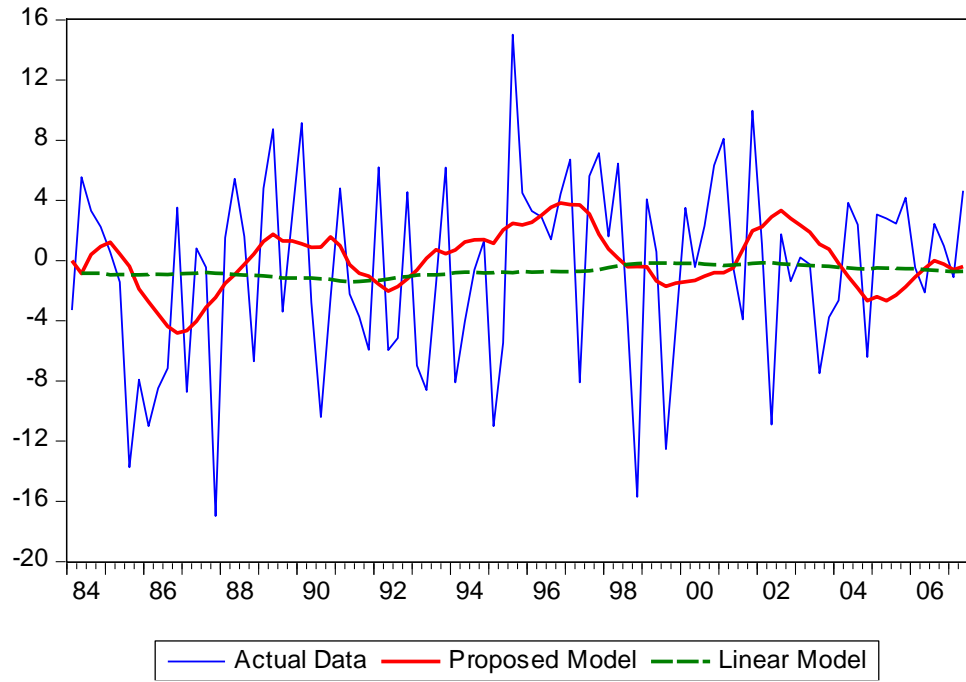
Because survey forecasts for the Germany mark changes to Euro from 1999Q1 and show large jump during this period, we drop the period 1998Q1-1999Q4 from the regressions for the Germany.

**FIGURE 1. In-Sample Prediction**

The graph plots the filtered series from the proposed model ( $\hat{\mu}_t$ ). The graph also plots the realized exchange rate changes as well as the fitted value from the OLS regression of exchange rate changes on lagged value of relative inflation and relative output gap. Data is quarterly, from 1984Q1-2007Q2.



### C. JAPAN



### D. UNITED KINGDOM

